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use the term "superlattice" to refer to an epitaxially grown structure in distinction to the term "multilayer" which is not epitaxially grown, although this usage does not have universal acceptance. The concept of superlattices was developed in the context of semiconductor MBE. In semiconductors the superlattice potential can strongly affect the quantum levels of the valence electrons. In this case the superlattice potential can be responsible for unique properties which are otherwise absent in the individual building block. In metallic magnetic superlattices, as we understand them so far, the situation is different. The basic magnetic properties such as magnetic anisotropies are mostly determined by the magnetic properties of the individual unit blocks (giant magnetic molecules) and by the strength of the exchange coupling through the non-magnetic interlayers. However, the existence of "magnetic" quantum well states in thin magnetic layers and multilayers has been reported recently and it is known that spin-split interface states exist at interfaces between magnetic and non-magnetic layers in superlattices. The importance of quantum well (resonance) states in the interlayer exchange coupling has recently been well established experimentally and theoretically. Zone folding effects have also been reported for magnetic excitations in magnetic superlattices, and the diffraction of X-rays or electrons into super-reflections defined by the multiple layering is a clearly a superlattice effect.

Studies of ultrathin magnetic structures are not limited to epitaxial structures only. In fact an essential part of this field is concerned with polycrystalline ultrathin structures with sharp interfaces grown by sputtering since a comparison of these systems yields insight into the morphology dependence of specific magnetic properties. Such structures can also exhibit properties which are comparable to those of structures grown epitaxially and in some cases the properties of sputtered structures are even enhanced with respect to those of the MBE grown structures – see for example Volume II Chap. 2 describing the magnetoresistivity of sputtered multilayer structures and Volume I Chap. 2 on the magnetic anisotropy behavior of sputtered films. Furthermore, from an application-oriented viewpoint, sputtered films are of strong interest because of the viability of producing samples commercially.

Finally, a word about units. This is a difficulty since many magneticians tend to use Gaussian units, partly because a large body of literature now exists which is written in these units, whereas many Europeans tend to use SI units automatically (or in some cases because they are required to). This issue is a particular concern for those entering the field. In writing this book it was first thought that it would be best to use one system of units only. But since it is by no means clear which units to use, it was decided that it would prove more educational if the book were to make use of both units and to include a conversion table between the two systems. For this reason, some sections are written in Gaussian units and others in SI, according to the authors' preferences. The reader is referred to the next section in which both systems of units are discussed and the conversion between them is given. While at first sight the reader may find it inconvenient to have to convert between units, we hope that after using this book he or she will agree that it is indeed necessary to do this and that anyone wishing to seriously read the literature in magnetism must be fully conversant with both systems.

1.2 Magnetism in SI Units and Gaussian Units

A.S. Arrott

Equations, units, dimensions and conversion factors are provided here as an aid to translation between the two languages of magnetism.

The late William Fuller Brown, Jr., founder of the field of micromagnetics, wrote a *Tutorial Paper on Dimensions and Units* [1.1], in which he said:

> *"If this seems a bit artibrary and confusing, bear in mind two principles: first, dimensions are the invention of man, and man is at liberty to assign them in any way he pleases, as long as he is consistent throughout any one interrelated set of calculations. Second, international committees arrive at their decisions by the same irrational procedures as do various IEEE committees that you have served on."*

To writers on the subject, Brown advises, "At all costs avoid tables: with them, you never know whether to multiply or divide." To the reader he consoles, "This terminology is quite arbitrary; don't take it too seriously, or you may get yourself into philosophical dilemmas."

The field of magnetism has been recently enriched by attachment to the world of surface science. This has created some demand for those in the field of magnetism to convert from the Gaussian system to the International System (SI). Workers in magnetism have been reluctant to use SI units for reasons that become apparent when the two systems are compared. Both systems have been said to be "a bit arbitrary and confusing." The arbitrariness adds to the confusion when translation is attempted. Both systems are self-consistent, but the translations exhibit apparent contradictions. Sympathy must be expressed for those surface scientists new to the field of magnetism. The following equations, units, dimensions and conversion factors are provided as an aid to translation between the two languages of magnetism.

The SI is based on the meter, kilogram, second and ampere, mksa. The Gaussian system is based on the centimeter, gram and second, cgs. It uses the electrostatic cgs units for electrical quantities and the electromagnetic cgs units for magnetic properties. In the Gaussian system, the velocity of light c appears in equations that have both magnetic and electrical quantities. In Maxwell's equations in Gaussian units the velocity of light multiplies each occurrence of the variable t whereever it appears, as in the expression $(1/c)\partial \boldsymbol{E}/\partial t$, or whereever it is implied by a hidden time derivative, as in the combination $(4\pi/c)I$. In the Gaussian system each source term is multiplied by 4π. In the SI system 4π is banished from the fundamental equations, but reappears when fields are calculated from sources.

Tables for converting between the two systems appear in many texts. It is harder to find direct comparisons of the equations of electricity and magnetism side by side as given here along with a discussion of units and dimensions. The presentation starts with the Lorentz force equation to introduce $\boldsymbol{E}$ and $\boldsymbol{B}$. This is followed by the electrical and then magnetic quantities leading to Maxwell's equations and the equation of motion for magnetism. The latter is a torque equation with the dimensions of energy density. As it is the same for both systems of units it appears in the center of the line:[1]

$$\frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \boldsymbol{B}_{\mathrm{eff}}, \tag{1.45}$$

where effective magnetic induction $\boldsymbol{B}_{\mathrm{eff}}$ is the vector gradient of the energy density $w(\boldsymbol{M})$ with respect to the components of the magnetization $\boldsymbol{M}$:

$$\boldsymbol{B}_{\mathrm{eff}} = -\nabla_{M} w(\boldsymbol{M}) + \lambda \boldsymbol{M}. \tag{1.46}$$

When the equation of motion is presented in terms of the magnetic field $\boldsymbol{H}$, the equations are different. The SI equations are put on the left and the Gaussian equations on the right.

In SI units: In Gaussian units:

$$\frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \mu_0 \boldsymbol{H}_{\mathrm{eff}}, \quad (1.62) \qquad \frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \boldsymbol{H}_{\mathrm{eff}}, \quad (1.64)$$

where where

$$\boldsymbol{H}_{\mathrm{eff}} = -\frac{\nabla_M w(\boldsymbol{M})}{\mu_0} + \lambda \boldsymbol{M}. \quad (1.63) \qquad \boldsymbol{H}_{\mathrm{eff}} = -\nabla_M w(\boldsymbol{M}) + \lambda \boldsymbol{M}. \quad (1.65)$$

The equations are explained as they are introduced. The units and dimensions of the quantities are listed on the left and right. The conversions between quantities in the two systems are given as equations in the center of the line. For example, some of the principle magnetic quantities are:

SI units	conversion	Gaussian units
$\boldsymbol{m}$ is in A m^2 = joule/tesla	10^{-3} J/T = 1 erg/G	$\boldsymbol{m}$ is in erg/G, emu
$\boldsymbol{B}$, $\mu_0\boldsymbol{M}$, and $\mu_0\boldsymbol{H}$ are in teslas, (T)	10^{-4} T = 1 G	$\boldsymbol{B}$ and $4\pi\boldsymbol{M}$ are in gausses, G
$\boldsymbol{M}$ is in A m^{-1}	1000 A m^{-1} = 1erg(G^{-1} cm^{-3})	$\boldsymbol{M}$ is in emu cm^{-3}
γ is in T^{-1}s^{-1}	10^4(T s)$^{-1}$ = 1 (G s)$^{-1}$	γ is in G^{-1} s^{-1}
$\boldsymbol{H}$ is in ampere(turns) m^{-1}	1000/4π A m^{-1} = 1 Oe	$\boldsymbol{H}$ is in oersteds, Oe
$\boldsymbol{\Phi}_B$ is in webers = T m^2	10^{-8} Wb = 1 maxwell	$\boldsymbol{\Phi}_B$ is in maxwells = G cm^2

[1] These equations appear again later in this section and the numbering refers to the sequence in which they are presented there.

In the Gaussian system $\boldsymbol{B}$, $\boldsymbol{H}$ and $\boldsymbol{M}$ have the same dimensions, but not the same units. This may seem strange to a visitor from SI space to Gaussian space. Indeed it can be a source of confusion, but note that torque and energy have the same dimensions, but are quoted in different units, e.g. N m and joule.

A general method of translating equations between the two systems is given in the final section. There are eight separate forms for the translation keys. The application of these to one particular equation is shown. The number of required steps illustrates why one might rather have all the pertinent equations side by side.

1.2.1 Equations of Electricity and Magnetism

in SI Units: {always on the left} and in Gaussian Units: {always on the right}

The Lorentz force equation defines the electric field $\boldsymbol{E}$ and the magnetic induction $\boldsymbol{B}$ in terms of the force $\boldsymbol{F}$ on a charge q and its velocity $\boldsymbol{v}$ with respect to the frame of reference:

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}, \quad (1.1) \qquad \boldsymbol{F}_{\mathrm{cgs}} = q_{\mathrm{esu}}\left(\boldsymbol{E}_{\mathrm{esu}} + \frac{\boldsymbol{v}_{\mathrm{cgs}} \times \boldsymbol{B}_{\mathrm{emu}}}{c_{\mathrm{cgs}}}\right). \quad (1.2)$$

$\boldsymbol{F}$ is in newton, N	10^{-5} N = 1 dyn	$\boldsymbol{F}_{\mathrm{cgs}}$ is in dyn
q is in coulomb, C	1/2997924580 C = 1 statcoulomb	q_{esu} is in statcoulomb
$\boldsymbol{v}$ is in m s^{-1}	0.01 m s^{-1} = 1 cm s^{-1}	$\boldsymbol{v}$ is in cm s^{-1}
$q\boldsymbol{v}$ is in ampere meter, A m		$q\boldsymbol{v}$ is in statampere cm
I is in ampere, A	$3.33564095 \times 10^{-10}$ A = 1 statampere	I_{esu} is in statampere
$\boldsymbol{E}$ is in N C^{-1}	29979.2458 N C^{-1} = 1 dyn statcoulomb^{-1}	$\boldsymbol{E}_{\mathrm{esu}}$ is in dyn statcoulomb^{-1}
1 N C^{-1} = 1 volt meter^{-1} = 1 V m^{-1}	299.792458 v = 1 statvolt	1 dyn/statcoulomb = 1 statvolt cm^{-1}
$\boldsymbol{B}$ is in teslas, T		$\boldsymbol{B}_{\mathrm{emu}}$ is in gauss, G
1 T = 1 N A^{-1}m^{-1} = J A^{-1}m^{-2}	10^{-4} T = 1 G	1 G = 1 dyn statcoulomb^{-1}

The dimensions [ ] of $\boldsymbol{E}$ are the same in the two systems but the dimensions of

B are not:

SI	Gaussian
$[\boldsymbol{E}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{distance}]^{-1}\,[\text{time}]^{-1}$	$[\boldsymbol{E}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{distance}]^{-1}\,[\text{time}]^{-1}$
$[\boldsymbol{B}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{distance}]^{-2}$	$[\boldsymbol{B}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{distance}]^{-1}\,[\text{time}]^{-1}$

Electrical quantities

Charge is a source of the electric field. In differential form the field has the total charge density ρ_T as a source:

$$\nabla\cdot\boldsymbol{E} = 4\pi k_e \rho_T\,, \tag{1.3}$$

$$\rho_T = \rho + \rho_P = \rho - \nabla\cdot\boldsymbol{P}_{dip}\,, \tag{1.4}$$

where $\boldsymbol{P}_{dip}$ is the dipole moment per unit volume. The subscript "dip" is dropped in the remainder of this chapter.

In SI:

$$k_e = \frac{1}{4\pi\varepsilon_0} = Kc^2\,, \tag{1.5}$$

$$K \equiv 10^{-7}\,\mathrm{N\,A^{-2}}\,, \tag{1.6}$$

$$\frac{1}{4\pi\varepsilon_0} = 8.98755179 \times 10^9\ \mathrm{V\,m\,C^{-1}}\,, \tag{1.7}$$

$$c \equiv 299792458\ \mathrm{m\,s^{-1}}\,, \tag{1.8}$$

$$\varepsilon_0 \nabla\cdot\boldsymbol{E} = \rho_T\,. \tag{1.9}$$

In esu and Gaussian units:

$$k_e = 1\,\frac{\text{statvolt cm}}{\text{statcoulomb}}\,. \tag{1.10}$$

In the Gaussian system

$$1\ \text{statvolt} = 1\ \text{statcoulomb cm}^{-1}\,,$$

so that k_e is usually considered to be dimensionless, but there are certain advantages in dimensional analysis if the dimensions of k_e are carried along.

$$\nabla\cdot\boldsymbol{E} = 4\pi\rho_T\,. \tag{1.11}$$

The dimensions of **P** are the same in the two systems:

$$\boxed{[\boldsymbol{P}] = [\text{current}]\,[\text{time}]\,[\text{length}]^{-2}\,.}$$

In both systems of units the electric dipole is $\boldsymbol{p} = q\boldsymbol{l}$ and the polarization **P** is the dipole moment per unit volume. The torque on an electric dipole $\boldsymbol{p}_{dip}$ in a field **E** is

$$\boldsymbol{T} = \boldsymbol{p}\times\boldsymbol{E}\,. \tag{1.12}$$

SI	Conversion	Gaussian
T is in joule rad^{-1} J m m^{-1}	10^{-7} J m m^{-1} = 1 erg cm cm^{-1}	**T** is in erg rad^{-1}

The energy W of a permanent electric dipole **p** in a field **E** is

$$W = -\boldsymbol{p}\cdot\boldsymbol{E}\,. \tag{1.13}$$

SI	Conversion	Gaussian
W is in J	10^{-7} J = 1 erg	W is in erg
ρ is in C m^{-3}		ρ is in statcoulomb cm^{-3}
p is in C m	3.33564095×10^{-12} C m = 1 statcoulomb cm	**p** is in statcoulomb cm
P is in C m^{-2}	3.33564095×10^{-6} C m^{-2} = 1 statcoulomb cm^{-2}	**P** is in statcoulomb cm^{-2}
E is in V m^{-1}		**E** is in statvolt cm^{-1}
$4\pi k_e\boldsymbol{P}$ is in V m^{-1}		$4\pi k_e\boldsymbol{P}$ is in statvolt cm^{-1}

SI	Gaussian
$[\boldsymbol{p}] = [\text{current}]\,[\text{time}]\,[\text{length}]$	$[\boldsymbol{p}] = [\text{current}]\,[\text{time}]\,[\text{length}]$
$[\boldsymbol{P}] = [\text{current}]\,[\text{time}]\,[\text{length}]^{-2}$	$[\boldsymbol{P}] = [\text{current}]\,[\text{time}]\,[\text{length}]^{-2}$
$[\boldsymbol{E}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{time}]^{-1}\,[\text{length}]^{-1}$	$[\boldsymbol{E}] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{time}]^{-1}\,[\text{length}]^{-1}$

$4\pi k_e\boldsymbol{P}$ appears as a source of **E**. The units and dimensions of $4\pi k_e\boldsymbol{P}$ and **E** are the same as each other in both systems. Unfortunately, the definition of the electric susceptibility is different in the two systems.

$$4\pi k_e\boldsymbol{P} = \chi_e\boldsymbol{E}\,, \tag{1.14}$$

$$\boldsymbol{P}_{esu} = \chi_e\boldsymbol{E}_{esu}\,. \tag{1.15}$$

The SI electric susceptibility is dimensionless. The Gaussian electric susceptibility is in statcoulombs/statvolt · cm, which is actually dimensionless.

$$\boxed{(\chi_e)_{SI} = 4\pi(\chi_e)_{Gaussian}\,.}$$

The translation between the systems is further confused by the contrary definitions of the electric flux density **D**.

$$4\pi k_e\boldsymbol{D} = \boldsymbol{E} + 4\pi k_e\boldsymbol{P}\,, \tag{1.16}$$

$$\boldsymbol{D} \equiv \varepsilon_0\boldsymbol{E} + \boldsymbol{P}\,, \tag{1.17}$$

$$\nabla\cdot\boldsymbol{D} = \rho\,, \tag{1.18}$$

$$\boldsymbol{D} \equiv \boldsymbol{E} + 4\pi k_e\boldsymbol{P}\,, \tag{1.19}$$

$$\boldsymbol{D} \equiv \boldsymbol{E} + 4\pi\boldsymbol{P}\,, \tag{1.20}$$

$$\nabla\cdot\boldsymbol{D} = 4\pi k_e\rho\,, \tag{1.21}$$

$$\nabla\cdot\boldsymbol{D} = 4\pi\rho\,. \tag{1.22}$$

SI		Gaussian
D is in $\mathrm{C\,m^{-2}}$	$3.33564095 \times 10^{-6}\,\mathrm{C\,m^{-2}} = 1\,\mathrm{statvolt\,cm^{-1}}$	D is in $\mathrm{statvolt\,cm^{-1}}$
$[D] = [\text{current}]\,[\text{time}]\,[\text{length}]^{-2}$		$[D] = [\text{energy}]\,[\text{current}]^{-1} \times [\text{time}]^{-1}\,[\text{length}]^{-1}$

Magnetic quantities

The magnetic induction $\boldsymbol{B}$ has no divergence. $\boldsymbol{E}$ and $\boldsymbol{B}$ are related by Faraday's laws of induction:

$$\nabla \cdot \boldsymbol{B} = 0, \quad (1.23) \qquad \nabla \cdot \boldsymbol{B} = 0, \quad (1.25)$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0, \quad (1.24) \qquad \nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0. \quad (1.26)$$

In Gaussian units the c appears explicitly in Faraday's Law. Maxwell introduced the analogous relation between curl $\boldsymbol{B}$ and the time derivative of $\boldsymbol{E}$ valid in free space:

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = 0, \quad (1.27) \qquad \nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = 0. \quad (1.28)$$

In the presence of current density $\boldsymbol{j}$ these two equations are:

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = 4\pi k_m \boldsymbol{j}, \quad (1.29) \qquad \nabla \times \boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = 4\pi k_m \boldsymbol{j}, \quad (1.32)$$

where (SI):

$$k_m = \frac{\mu_0}{4\pi} = 10^{-17}\,\mathrm{N\,A^{-2}}, \quad (1.30)$$

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}} = 4\pi \times 10^{-7}\,\mathrm{henry\,m^{-1}} \quad (1.31)$$

where (Gaussian):

$$k_m = 1/c, \quad (1.33)$$

$$c = 2.99792458 \times 10^{10}\,\mathrm{cm\,s^{-1}}. \quad (1.34)$$

In SI units the c's are hidden in the expression for the sources of curl $\boldsymbol{B}$, which in differential form are current densities including the polarization currents and the amperian currents as well as the true current density $\boldsymbol{j}$:

$$\nabla \times \boldsymbol{B} - \varepsilon_0 \mu_0 \frac{\partial \boldsymbol{E}}{\partial t} = \mu_0\left(\boldsymbol{j} + \frac{\partial \boldsymbol{P}}{\partial t} + \nabla \times \boldsymbol{M}\right), \quad (1.35)$$

$$\nabla \times \boldsymbol{E} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\left(\boldsymbol{j} + \frac{\partial \boldsymbol{P}}{\partial t} + c\nabla \times \boldsymbol{M}\right). \quad (1.36)$$

There is a $1/c^2$ in the SI equation from the combination $\mu_0 \varepsilon_0$. The c appears in front of the curl $\boldsymbol{M}$ term in Gaussian units to cancel the $1/c$ outside the bracket, because this term is measured in emu. The magnetization $\boldsymbol{M}$ in both systems is the magnetic moment per unit volume. In both cases the magnetic moment $\boldsymbol{m}$ is defined in terms of a current I in a loop with a finite area A and normal $\hat{\boldsymbol{n}}$:

$$\boldsymbol{m} = IA\,\hat{\boldsymbol{n}}, \quad (1.37) \qquad \boldsymbol{m} = \frac{IA}{c}\hat{\boldsymbol{n}}. \quad (1.38)$$

The c appears in Gaussian units because I is measured in esu and $\boldsymbol{m}$ in emu. NB The magnetic moment of the electron is the Bohr magneton with a quantum electrodynamic correction. The Bohr magneton is:

$$\mu_B = \frac{eh}{4\pi m_e}, \quad (1.39) \qquad \mu_B = \frac{eh}{4\pi c m_e}. \quad (1.40)$$

Some fundamental constants in the two systems are:

SI	Gaussian
Bohr magneton: $\mu_B = 9.2740154 \times 10^{-24}\,\mathrm{J\,T^{-1}}$	Bohr magneton: $\mu_B = 9.2740154 \times 10^{-21}\,\mathrm{erg\,G^{-1}}$
electron moment: $\mu_e = 9.2750909 \times 10^{-24}\,\mathrm{J\,T^{-1}}$	electron moment: $\mu_e = 9.2750909 \times 10^{-21}\,\mathrm{erg\,G^{-1}}$
Planck's constant: $h = 6.6260755 \times 10^{-34}\,\mathrm{J\,s}$	Planck's constant: $h = 6.6260755 \times 10^{-27}\,\mathrm{erg\,s}$
electron mass: $m_e = 9.1093897 \times 10^{-31}\,\mathrm{kg}$	electron mass: $m_e = 9.1093897 \times 10^{-28}\,\mathrm{g}$

The magnetic induction produces a torque on a magnetic moment, which follows from the Lorentz force equations (1.1, 2). In both cases:

$$\boldsymbol{T} = \boldsymbol{m} \times \boldsymbol{B}. \quad (1.41)$$

The energy W_m of a magnetic moment follows from the torque equation if the zero of energy is taken from the position where $\boldsymbol{m}$ is perpendicular to $\boldsymbol{B}$:

$$W_m = -\boldsymbol{m} \cdot \boldsymbol{B}. \quad (1.42)$$

The dynamic response of a magnetic moment to a torque is determined by the ratio of the magnetic moment to the angular momentum, called the gyromagnetic ratio γ. Again the equations in the two systems differ by the factor c:

$$\gamma = -g\frac{|e|}{2m_e}, \quad (1.43) \qquad \gamma = -g\frac{|e|}{2cm_e}. \quad (1.44)$$

The dynamic equation for a magnetic moment $\boldsymbol{m}$ in the presence of a magnetic induction $\boldsymbol{B}$ in both systems is:

$$\frac{1}{\gamma}\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{m} \times \boldsymbol{B}. \tag{1.45}$$

There are also nonmagnetic sources of torque on a magnetic moment, e.g. exchange torques and spin-orbit interactions. These are included in the equation of motion using an effective magnetic induction $\boldsymbol{B}_{\text{eff}}$, which can include any arbitrary vector parallel to $\boldsymbol{m}$. When expressed in terms of $\boldsymbol{M}$, the equation of motion becomes:

$$\frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \boldsymbol{B}_{\text{eff}}, \tag{1.46}$$

where $\boldsymbol{B}_{\text{eff}}$ is the vector gradient of the energy density $w(\boldsymbol{M})$ with respect to the magnetization components:

$$\boldsymbol{B}_{\text{eff}} = -\nabla_M w(\boldsymbol{M}) + \lambda \boldsymbol{M}. \tag{1.47}$$

Because $\boldsymbol{M} \times \boldsymbol{M} = 0$, one can include any $\lambda \boldsymbol{M}$ into (1.46), where λ is chosen for convenience and can vary with position. Much confusion results in comparing various authors, because $\boldsymbol{B}_{\text{eff}}$ is not uniquely defined. This carries over when the magnetic field $\boldsymbol{H}$ is introduced into Maxwell's equations. The magnetic field $\boldsymbol{H}$ is defined as that part of $\boldsymbol{B}$ which is not directly the contribution of $\boldsymbol{M}$ to $\boldsymbol{B}$. The contribution of $\boldsymbol{M}$ to $\boldsymbol{B}$ is different in the two systems. It is $\mu_0 \boldsymbol{M}$ in SI and $4\pi \boldsymbol{M}$ in the Gaussian system. To keep the units of $\boldsymbol{H}$ and $\boldsymbol{M}$ the same in the SI system, $\boldsymbol{H}$ is defined with $\boldsymbol{B}$ divided by μ_0. The distinction between $\boldsymbol{H}$ and $\boldsymbol{B}$ is not so reinforced in Gaussian units, where they have different names for the units, but the two quantities have the same dimensions, in fact the same dimensions as $\boldsymbol{M}$ and $4\pi \boldsymbol{M}$. The definitions of $\boldsymbol{H}$ are:

$$\boldsymbol{H} \equiv \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{M}. \tag{1.48}$$

$$\boldsymbol{H} \equiv \boldsymbol{B} - 4\pi \boldsymbol{M}. \tag{1.49}$$

$\boldsymbol{B}$, $\mu_0 \boldsymbol{M}$ and $\mu_0 \boldsymbol{H}$ are in teslas, T	$10^{-4}\,\text{T} = 1\,\text{G}$	$\boldsymbol{B}$ is in gausses, G
$\boldsymbol{H}$ is in A (turns) m^{-1}	$1000/4\pi\ \text{A m}^{-1} = 1\,\text{Oe}$	$\boldsymbol{H}$ is in oersteds, Oe {1 Oe = 1 G}
$\boldsymbol{M}$ is in A m^{-1}	$1000\ \text{A m}^{-1} = 1\ \text{emu cm}^{-3}$	$\boldsymbol{M}$ is in ergs $(\text{G}^{-3}\,\text{cm}^{-3})$ or emu cm^{-3} {1 emu cm^{-3} = 1 erg $(\text{G}^{-3}\,\text{cm}^{-3})$ = 1 Oe = 1 G}

$[H] = [M] = [\text{current}]\,[\text{distance}]^{-1}$

$[B] = [H] = [M] = [\text{energy}] \times [\text{current}]^{-1}\,[\text{time}]^{-1}\,[\text{distance}]^{-1}$

If both $\boldsymbol{H}$ and $\boldsymbol{M}$ are measured in amperes/meters in SI and both $\boldsymbol{H}$ and $\boldsymbol{M}$ are measured in oersteds in Gaussian units, then the conversion from amperes/meter to oersteds depends on whether it is $\boldsymbol{H}$ or $\boldsymbol{M}$ that is being converted! This confusion is avoided by not using Oe for $\boldsymbol{M}$. If $\boldsymbol{M}$ is quoted in Oe, first convert to emu cm^{-3} and then convert to A m^{-1}.

Some confusion can be avoided also if one uses the magnetic polarization, either $\mu_0 \boldsymbol{M}$ in tesla or $4\pi \boldsymbol{M}$ in gauss, to report data. Similarly the magnetic field can be reported as $\mu_0 \boldsymbol{H}$ in tesla, particularly for the applied field.

The introduction of $\boldsymbol{H}$ is particularly useful in ferromagnetism. It plays an important role in magnetostatics, where $\operatorname{div} \boldsymbol{M}$ provides a source of $\boldsymbol{H}$. The magnetic charge density is defined as proportional to $-\operatorname{div} \boldsymbol{M}$, but the proportionality factor is different in the two systems:

$$\nabla \cdot \boldsymbol{H} = -\nabla \cdot \boldsymbol{M} = \rho_{\text{m}}, \tag{1.50}$$

$$\nabla \cdot \boldsymbol{H} = -4\pi \nabla \cdot \boldsymbol{M} = \rho_{\text{m}}. \tag{1.51}$$

The factor of 4π enters SI in the expressions for the magnetic field and magnetic induction arising from a magnetic moment in the absence of material media, expressed in terms of the magnetic moment $\boldsymbol{m}$ and the unit vector $\hat{\boldsymbol{r}}$:

$$\frac{1}{\mu_0}\boldsymbol{B} = \boldsymbol{H} = \frac{3(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{m}}{4\pi r^3}, \tag{1.52}$$

$$\boldsymbol{B} = \boldsymbol{H} = \frac{3(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{m}}{r^3}. \tag{1.53}$$

Maxwell's equations are often written in terms of $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{B}$, and $\boldsymbol{H}$, suppressing the roles of $\boldsymbol{P}$ and $\boldsymbol{M}$. These are particularly simple in SI units, containing neither of the constants ε_0 or μ_0 and none of the 4π's or c's that appear in the Gaussian version of Maxwell's equations:

$$\nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{j}, \tag{1.54}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{1.55}$$

$$\nabla \cdot \boldsymbol{D} = \rho, \tag{1.56}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{1.57}$$

$$\nabla \times \boldsymbol{H} - \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} = \frac{4\pi}{c}\boldsymbol{j}, \tag{1.58}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{1.59}$$

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho, \tag{1.60}$$

$$\nabla \cdot \boldsymbol{B} = 0. \tag{1.61}$$

The equations of motion (1.45, 46) in terms of $\boldsymbol{H}$ include a factor of μ_0 in the SI units. As above, in (1.43, 44) and [44], the gyromagnetic ratio is defined differently in the two systems.

$$\frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \mu_0 \boldsymbol{H}_{\text{eff}}, \quad (1.62) \qquad \frac{1}{\gamma}\frac{\partial \boldsymbol{M}}{\partial t} = \boldsymbol{M} \times \boldsymbol{H}_{\text{eff}}, \quad (1.64)$$

where $\qquad\qquad$ where

$$\boldsymbol{H}_{\text{eff}} = -\frac{\nabla_M w(\boldsymbol{M})}{\mu_0} + \lambda \boldsymbol{M}. \quad (1.63) \qquad \boldsymbol{H}_{\text{eff}} = -\nabla_M w(\boldsymbol{M}) + \lambda \boldsymbol{M}. \quad (1.65)$$

Because the magnetic field differs from the magnetic induction only by a vector along $\boldsymbol{M}$, it does not matter whether one talks about an effective field or an effective induction, but in SI units it is necessary to remember that the torque equation contains the factor μ_0. Derivatives of energy densities with respect to $\boldsymbol{M}$ produce magnetic inductions. In Gaussian units these also have the dimensions of magnetic field. In SI units the derivatives of energies with respect to $\boldsymbol{M}$ must be divided by μ_0 to make them effective fields rather than effective inductions.

The magnetic contribution to the energy density w is:

$$w = \frac{\boldsymbol{B}\cdot\boldsymbol{H}}{2}; \quad (1.66) \qquad w = \frac{\boldsymbol{B}\cdot\boldsymbol{H}}{8\pi}. \quad (1.67)$$

The magnetostatic self energy of an aggregation of magnetic moments, excluding the self energy of the individual moments, is:

$$W = -\frac{\mu_0}{2}\sum_i \boldsymbol{m}_i \boldsymbol{H}_i' = -\frac{1}{2}\sum_i \boldsymbol{m}_i \boldsymbol{B}_i', \quad (1.68) \qquad W = -\frac{1}{2}\sum_i \boldsymbol{m}_i \boldsymbol{H}_i', \quad (1.70)$$

where $\boldsymbol{H}_i'$ is the demagnetizing field from all the other moments: $\qquad$ where $\boldsymbol{H}_i'$ is the demagnetizing field from all the other moments:

$$\boldsymbol{H}_i' = \sum_{j\neq i} \frac{3(\boldsymbol{m}_j\cdot\hat{\boldsymbol{r}}_{ij})\hat{\boldsymbol{r}}_{ij} - \boldsymbol{m}_j}{4\pi r_{ij}^3} = \frac{\boldsymbol{B}_i'}{\mu_0}. \quad (1.69) \qquad \boldsymbol{H}_i' = \sum_{j\neq i} \frac{3(\boldsymbol{m}_j\cdot\hat{\boldsymbol{r}}_{ij})\hat{\boldsymbol{r}}_{ij} - \boldsymbol{m}_j}{r_{ij}^3} = \boldsymbol{B}_i'. \quad (1.71)$$

For a uniformly magnetized ellipsoid the demagnetizing field is uniform and proportional to the magnetic moment per unit volume. For a uniformly magnetized sphere, the demagnetizing field $\boldsymbol{H}_{\text{d}}$ is:

$$\boldsymbol{H}_{\text{d}} = -\boldsymbol{M}/3. \quad (1.72) \qquad \boldsymbol{H}_{\text{d}} = -4\pi\boldsymbol{M}/3. \quad (1.73)$$

For a slab magnetized perpendicularly to the surface, the demagnetizing field $\boldsymbol{H}_{\text{d}}$ is:

$$\boldsymbol{H}_{\text{d}} = -\boldsymbol{M}. \quad (1.74) \qquad \boldsymbol{H}_{\text{d}} = -4\pi\boldsymbol{M}. \quad (1.75)$$

The factor of 4π also appears in the conversion of magnetic susceptibility between the two systems. In both systems $\boldsymbol{M} = \chi\boldsymbol{H}$ defines the volume susceptibility. This is a dimensionless ratio if $\boldsymbol{M}$ and $\boldsymbol{H}$ are in the same units, but

$$\boxed{\chi_{\text{SI}} = 4\pi\chi_{\text{Gaussian}}.} \quad (1.76)$$

One could say that the units of χ_{SI} are turn^{-1}, and the units of χ_{Gaussian} are erg/(G Oe cm^3).

The magnetic flux is defined in the same way in each system:

$$\Phi_B = \int_{\text{area}} \boldsymbol{B}\cdot\hat{\boldsymbol{n}}\,\mathrm{d}A, \quad (1.77)$$

$$\boxed{\begin{array}{c}\Phi_B \text{ is in webers,}\\ 1\ \text{Wb} = 1\ \text{T m}^2\end{array}} \qquad \boxed{10^{-8}\ \text{Wb} = 1\ \text{maxwell.}} \qquad \boxed{\begin{array}{c}\Phi_B \text{ is in maxwells}\\ = \text{G cm}^2.\end{array}}$$

Faraday's law of induction in integral form is:

$$\oint_{\text{loop}} \boldsymbol{E}\cdot\hat{\boldsymbol{s}}\,\mathrm{d}s = -\frac{\partial}{\partial t}\Phi_B. \quad (1.78) \qquad \oint_{\text{loop}} \boldsymbol{E}\cdot\hat{\boldsymbol{s}}\,\mathrm{d}s = -\frac{1}{c}\frac{\partial}{\partial t}\Phi_B. \quad (1.79)$$

1.2.2 Translation Keys

To translate from **equations** in the Gaussian system to **equations** in SI, the following table of translation equations provides the keys [1.2]. Solve the appropriate equation for the starred variable, substitute for each quantity in the Gaussian system, and then clean up using the relation $\varepsilon_0\mu_0 c^2 = 1$ to remove the c's which are hidden in SI. To translate from **equations in SI** to **equations** in the Gaussian system, solve for the unstarred variable, substitute for each quantity in SI, and then remove the ε_0 and μ_0 factors using $\varepsilon_0\mu_0 c^2 = 1$. The fact that eight different translation equations are needed accounts for much of the confusion that attends the subject.

$$\sqrt{4\pi\varepsilon_0} = \frac{E^*}{E} = \frac{V^*}{V} = \frac{Q}{Q^*} = \frac{\rho}{\rho^*} = \frac{j}{j^*} = \frac{I}{I^*} = \frac{P}{P^*}; \quad (1.80)$$

$$\sqrt{4\pi\mu_0} = \frac{H^*}{H}; \qquad \sqrt{\frac{4\pi}{\varepsilon_0}} = \frac{D^*}{D}; \qquad \sqrt{\frac{4\pi}{\mu_0}} = \frac{B^*}{B} = \frac{M}{M^*} = \frac{\gamma}{\gamma^*} \quad (1.81)$$

$$1 = \frac{x^*}{x} = \frac{t^*}{t} = \frac{m^*}{m} = \frac{c^*}{c} = \frac{F^*}{F} = \frac{W^*}{W}; \qquad 4\pi = \frac{\chi_{\text{e}}}{\chi_{\text{e}}^*} = \frac{\chi_{\text{m}}}{\chi_{\text{m}}^*}; \qquad \varepsilon_0\mu_0 c^2 = 1. \quad (1.82)$$

For example:
start with

$$\gamma = -g\frac{|e|}{2m_e}, \tag{1.83}$$

look in the table for the transfer relation of γ and γ^*:

$$\gamma = \sqrt{\frac{4\pi}{\mu_0}}\gamma^* \tag{1.84}$$

and the transfer relation of charge:

$$|e| = |e^*|\sqrt{4\pi\varepsilon_0}. \tag{1.85}$$

Substitute these, $g = g^*$ and $m = m^*$ in (1.83) to obtain:

$$\sqrt{\frac{4\pi}{\mu_0}}\gamma^* = -g^*\frac{|e^*|\sqrt{4\pi\varepsilon_0}}{2m_e^*}, \tag{1.86}$$

which using $c^2\mu_0\varepsilon_0 = 1$ and $c = c^*$ simplifies to:

$$\gamma^* = -g^*\frac{|e^*|}{2c^*m_e^*}. \tag{1.87}$$

or start with

$$\gamma^* = -g^*\frac{|e^*|}{2c^*m_e^*}, \tag{1.88}$$

look in the table for the transfer relation of γ and γ^*:

$$\gamma^* = \sqrt{\frac{\mu_0}{4\pi}}\gamma \tag{1.89}$$

and the transfer relation of charge:

$$|e^*| = \frac{1}{\sqrt{4\pi\varepsilon_0}}|e| \tag{1.90}$$

Substitute these, $g = g^*$, $c^* = c$ and $m^* = m$ in (1.88) to obtain:

$$\sqrt{\frac{\mu_0}{4\pi}}\gamma = -g\frac{|e|}{2cm_e\sqrt{4\pi\varepsilon_0}}, \tag{1.91}$$

which using $c^2\mu_0\varepsilon_0 = 1$ simplifies to:

$$\gamma = -g\frac{|e|}{2m_e}. \tag{1.92}$$

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References

Section 1.2

1.1 William Fuller Brown, Jr.: IEEE Trans. Mag. MAG-20, 112–117 (1984)
1.2 *Symbols, Units, Nomenclature and Fundamental Constants in Physics*, 1987 Revision, Prepared by E. Richard Cohen and Pierre Giacomo for the International Union of Pure and Applied Physics: Physica **146A**, 1–68 (1987)