Chapter nine

Transient Behaviour

Objectives

When you have studied the material in this chapter, you should be able to:

- **explain concepts such as steady-state response, transient response and total response as they apply to electronic circuits**
- **describe the transient behaviour of simple *RC* and *RL* circuits**
- **predict the transient response of a generalised first-order system from a knowledge of its initial and final values**
- **sketch increasing or decreasing waveforms and identify their key characteristics**
- **describe the output of simple *RC* and *RL* circuits in response to a square-wave input**
- **outline the transient behaviour of various forms of second-order systems.**

9.1 Introduction

In earlier chapters, we looked at the behaviour of circuits in response to either fixed DC signals or constant AC signals. Such behaviour is often referred to as the **steady-state response** of the system. Now it is time to turn our attention to the performance of circuits before they reach this steady-state condition: for example, how the circuits react when a voltage or current source is initially turned on or off. This is referred to as the **transient response** of the circuit.

We will begin by looking at simple *RC* and *RL* circuits and then progress to more complex arrangements.

9.2 Charging of capacitors and energising of inductors

9.2.1 Capacitor charging

Figure 9.1(a) shows a circuit that charges a capacitor C from a voltage source V through a resistor R. The capacitor is assumed to be initially uncharged, and the switch in the circuit is closed at time $t = 0$.

When the switch is first closed the charge on the capacitor is zero, and therefore the voltage across it is also zero. Thus all the applied voltage is across the resistor, and the initial current is given by V/R. As this current flows into the capacitor the charge on it builds and the voltage across it increases. As the voltage across the capacitor increases, the voltage across the resistor decreases, causing the current in the circuit to fall. Gradually, the voltage across the capacitor increases until it is equal to the applied voltage, and the current goes to zero. We can understand this process more fully by

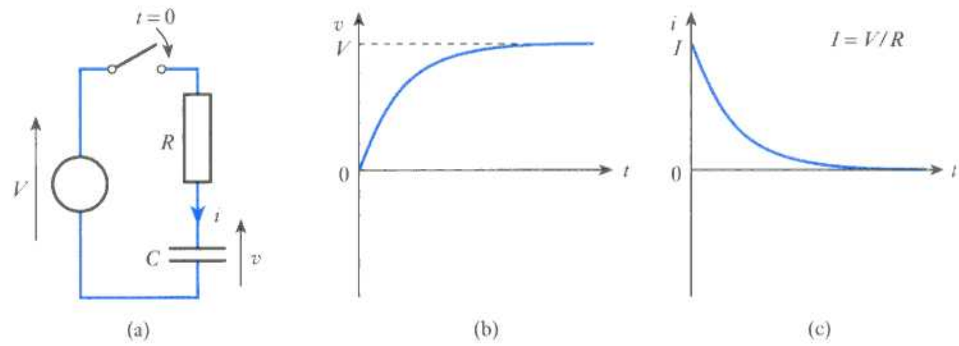


Figure 9.1 Capacitor charging.

deriving expressions for the voltage across the capacitor v and the current flowing into the capacitor i.

Applying Kirchhoff's voltage law to the circuit of Figure 9.1(a), we see that

$$iR + v = V$$

From Chapter 4, we know that the current in a capacitor is related to the voltage across it by the expression

$$i = C\frac{dv}{dt}$$

therefore, substituting,

$$CR\frac{dv}{dt} + v = V$$

This is a first-order differential equation with constant coefficients and is relatively easy to solve. First we rearrange the expression to give

$$\frac{dv}{dt} = \frac{V - v}{CR}$$

and then again to give

$$\frac{dt}{CR} = \frac{dv}{V - v}$$

Integrating both sides then gives

$$\frac{t}{CR} = -\ln(V - v) + A$$

where A is the constant of integration.

In this case we know (from our assumption that the capacitor is initially uncharged) that when $t = 0$, $v = 0$. Substituting this into the previous equation gives

$$\frac{0}{CR} = -\ln(V - 0) + A$$

$$A = \ln V$$

Therefore

$$\frac{t}{CR} = -\ln(V - v) + \ln V = \ln\frac{V}{V - v}$$

and

$$e^{t/CR} = \frac{V}{V - v}$$

Finally, rearranging we have

$$v = V(1 - e^{-t/CR}) \tag{9.1}$$

From this expression, we can also derive an expression for the current I, since

$$i = C\frac{dv}{dt} = CV\frac{d}{dt}(1 - e^{-t/CR}) = \frac{V}{R}e^{-t/CR}$$

We noted earlier that at $t = 0$ the voltage across the capacitor is zero and the current is given by V/R. If we call this initial current I, then our expression for the current becomes

$$i = Ie^{-t/CR} \tag{9.2}$$

In Equations 9.1 and 9.2, you will note that the exponential component contains the term t/CR. You will recognise CR as the time constant T of the circuit, and thus t/CR is equal to t/T and represents time as a fraction of the time constant. For this reason, it is common to give these two equations in a more general form, replacing CR by T:

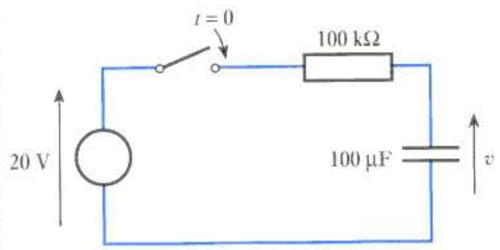
$$v = V(1 - e^{-t/\mathrm{T}}) \tag{9.3}$$

$$i = Ie^{-t/\mathrm{T}} \tag{9.4}$$

From Equations 9.3 and 9.4, it is clear that in the circuit of Figure 9.1(a) the voltage rises with time, while the current falls exponentially. These two waveforms are shown in Figures 9.1(b) and 9.1(c).

Example 9.1

The switch in the following circuit is closed at $t = 0$. Derive an expression for the output voltage v after this time and hence calculate the voltage on the capacitor at $t = 25$ s.



The time constant of the circuit $\mathrm{T} = CR = 100 \times 10^3 \times 100 \times 10^{-6} = 10$ s. From Equations 9.3

$$v = V(1 - e^{-t/\mathrm{T}})$$

$$= 20(1 - e^{-t/10})$$

At $t = 25$ s

$$v = 20(1 - e^{-25/10})$$

$$= 18.36 \text{ V}$$

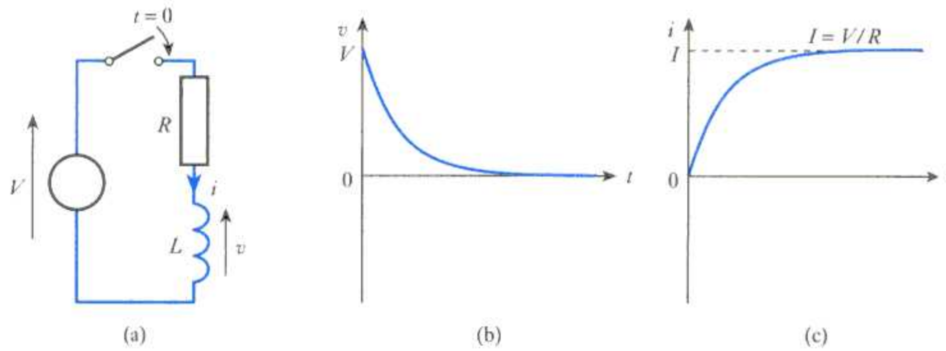


Figure 9.2 Inductor energising.

9.2.2 Inductor energising

Figure 9.2(a) shows a circuit that energises an inductor L using a voltage source V and a resistor R. The circuit is closed at time $t = 0$, and before that time no current flows in the inductor.

When the switch is first closed the current in the circuit is zero, since the nature of the inductor prevents the current from changing instantly. If the current is zero there is no voltage across the resistor, so all the applied voltage appears across the inductor. The applied voltage causes the current to increase, producing a voltage drop across the resistor and reducing the voltage across the inductor. Eventually, the voltage across the inductor falls to zero and all the applied voltage appears across the resistor, producing a steady current of V/R. As before, it is interesting to look at expressions for v and i.

Applying Kirchhoff's voltage law to the circuit of Figure 9.2(a), we see that

$$iR + v = V$$

From Chapter 5, we know that the voltage across an inductor is related to the current through it by the expression

$$v = L\frac{di}{dt}$$

therefore, substituting,

$$iR + L\frac{di}{dt} = V$$

This first-order differential equation can be solved in a similar manner to that derived for capacitors above. This produces the equations

$$v = Ve^{-Rt/L} \tag{9.5}$$

$$i = I(1 - e^{-Rt/L}) \tag{9.6}$$

where I represents the final (maximum) current in the circuit and is equal to V/R. In Equations 9.5 and 9.6, you will note that the exponential component contains the term Rt/L. Now L/R is the time constant T of the circuit, thus Rt/L is equal to t/T. We can therefore rewrite these two equations as

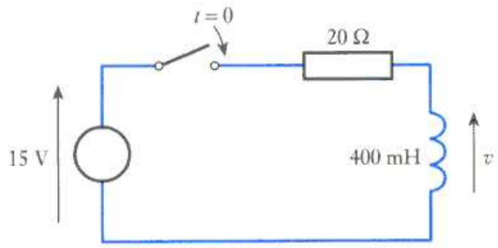
$$v = Ve^{-t/\mathsf{T}} \tag{9.7}$$

$$i = I(1 - e^{-t/\mathsf{T}}) \tag{9.8}$$

The forms of v and i are shown in Figures 9.2(b) and 9.2(c). You might like to compare these figures with Figures 9.1(b) and 9.1(c) for a charging capacitor. You might also like to compare Equations 9.7 and 9.8, which describe the energising of an inductor, with Equations 9.3 and 9.4, which we derived earlier to describe the charging of a capacitor.

Example 9.2

An inductor is connected to a 15 V supply as shown below. How long after the switch is closed will the current in the coil reach 300 mA?



The time constant of the circuit $\text{T} = L/R = 0.4 \div 20 = 0.02$ s. The final current I is given by $V/R = 15/20 = 750$ mA.

From Equations 9.8

$$i = I(1 - e^{-t/\text{T}})$$

$$300 = 750(1 - e^{-t/0.02})$$

which can be evaluated to give

$$t = 10.2 \text{ ms}$$

9.3 Discharging of capacitors and de-energising of inductors

The charging of a capacitor or the energising of an inductor stores energy in that component that can be used at a later time to produce a current in a circuit. In this section, we look at the voltages and currents associated with this process.

9.3.1 Capacitor discharging

In order to look at the discharging of a capacitor, first we need to charge it up. Figure 9.3(a) shows a circuit in which a capacitor C is initially connected

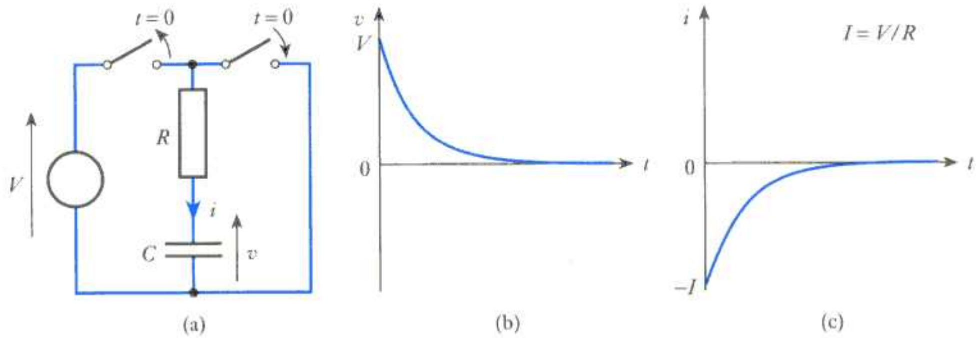


Figure 9.3 Capacitor discharging.

to a voltage source V and is then discharged though a resistor R. The discharge is initiated at $t = 0$ by opening one switch and closing another. In this diagram, the defining direction of the current i is *into* the capacitor, as in Figure 9.1(a), but clearly during the discharge process charge flows *out* of the capacitor, so i is negative.

The charged capacitor produces an electromotive force that drives a current around the circuit. Initially, the voltage across the capacitor is equal to the voltage of the source used to charge it (V), so the initial current is equal to V/R. However, as charge flows out of the capacitor its voltage decreases and the current falls. v and i can be determined in a similar manner to that used above for the charging arrangement. Applying Kirchhoff's voltage law to the circuit gives

$$iR + v = 0$$

giving

$$CR\frac{dv}{dt} + v = 0$$

Solving this as before leads to the expressions

$$v = Ve^{-t/CR} = Ve^{-t/T} \tag{9.9}$$

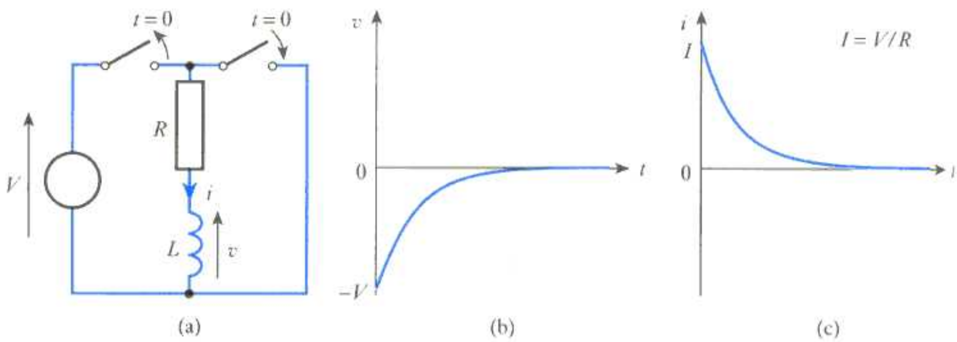
$$i = -Ie^{-t/CR} = -Ie^{-t/T} \tag{9.10}$$

As before, the voltage and current have an exponential form, and these are shown in Figures 9.3(b) and 9.3(c). Note that if i were defined in the opposite direction (as the current flowing out of the capacitor) then the polarity of the current in Figure 9.3(c) would be reversed. In this case, both the voltage and current would be represented by similar decaying exponential waveforms.

9.3.2 Inductor de-energising

In the circuit of Figure 9.4(a), a voltage source is used to energise an inductor by passing a constant current through it. At time $t = 0$ one switch is closed and the other is opened, so the energy stored in the inductor is now dissipated in the resistor. Since the current in an inductor cannot change instantly, initially the current flowing in the coil is maintained. To do this the inductor produces an electromotive force that is in the opposite direction to the potential created across it by the voltage source. With time, the energy

Figure 9.4 Inductor de-energising.



stored in the inductor is dissipated and the e.m.f. decreases and the current falls.

As before, v and i can be determined by applying Kirchhoff's voltage law to the circuit. This gives

$$iR + v = 0$$

and thus

$$iR + L\frac{di}{dt} = 0$$

Solving this as before leads to the expressions

$$v = -Ve^{-Rt/L} = -Ve^{-t/T} \quad (9.11)$$

$$i = Ie^{-Rt/L} = Ie^{-t/T} \quad (9.12)$$

As before, the voltage and current have an exponential form, and these are shown in Figures 9.4(b) and 9.4(c).

9.4 Generalised response of first-order systems

We have seen in Sections 9.2 and 9.3 that circuits containing resistance and either capacitance *or* inductance can be described by first-order differential equations. For this reason, such circuits are described as **first-order systems**. We have also seen that the transient behaviour of these circuits produces voltages and currents that change exponentially with time. However, although the various waveforms are often similar in form, they are not identical for different circuits. Fortunately, there is a simple method of determining the response of such systems to sudden changes in their environment.

9.4.1 Initial and final value formulae

Increasing and decreasing exponential waveforms (for either voltage or current) can be found from the expressions

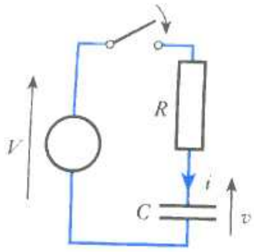
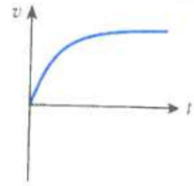
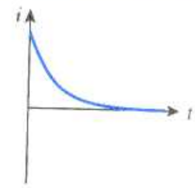
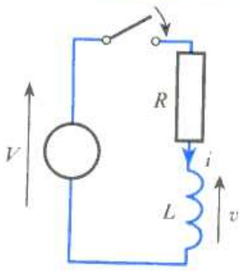
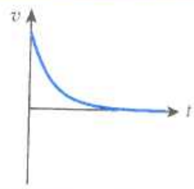
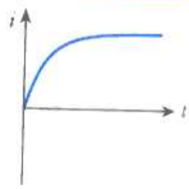
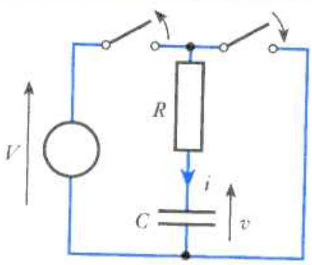
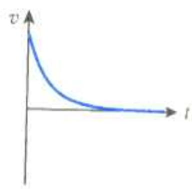
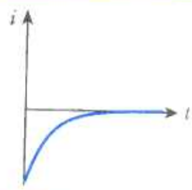
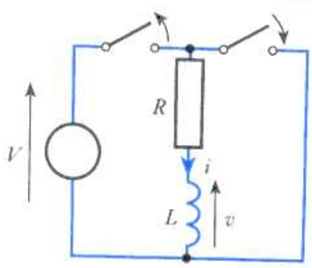
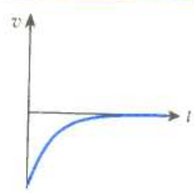
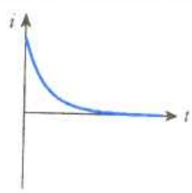
$$v = V_f + (V_i - V_f)e^{-t/T} \quad (9.13)$$

$$i = I_f + (I_i - I_f)e^{-t/T} \quad (9.14)$$

where V_i and I_i are the *initial* values of the voltage and current, and V_f and I_f are the *final* values. The first element in these two expressions represents the steady-state response of the circuit, which lasts indefinitely. The second element represents the transient response of the circuit. This has a magnitude determined by the step change applied to the circuit, and it decays at a rate determined by the time constant of the arrangement. The combination of the steady-state and the transient response gives the **total response** of the circuit. To see how these formulae can be used, Table 9.1 shows them applied to the circuits discussed in Sections 9.2 and 9.3.

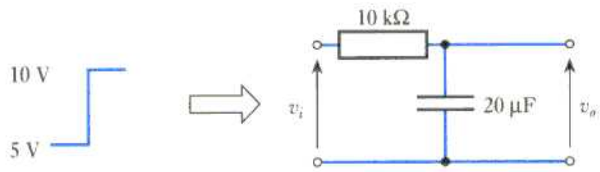
The **initial and final value formulae** are not restricted to situations where a voltage or current changes to, or from, zero. They can be used wherever there is a step change in the voltage or current applied to a first-order network. This is illustrated in Example 9.3.

Table 9.1 Transient response of first-order systems.

	$V_i = 0 \quad V_f = V$ $I_i = V/R = I \quad I_f = 0$ $T = CR$	$v = V_f + (V_i - V_f)e^{-t/T}$ $= V + (0 - V)e^{-t/T}$ $= V(1 - e^{-t/T})$	
		$i = I_f + (I_i - I_f)e^{-t/T}$ $= 0 + (I - 0)e^{-t/T}$ $= Ie^{-t/T}$	
	$V_i = V \quad V_f = 0$ $I_i = 0 \quad I_f = V/R = I$ $T = L/R$	$v = V_f + (V_i - V_f)e^{-t/T}$ $= 0 + (V - 0)e^{-t/T}$ $= Ve^{-t/T}$	
		$i = I_f + (I_i - I_f)e^{-t/T}$ $= I + (0 - I)e^{-t/T}$ $= I(1 - e^{-t/T})$	
	$V_i = V \quad V_f = 0$ $I_i = -V/R = -I \quad I_f = 0$ $T = CR$	$v = V_f + (V_i - V_f)e^{-t/T}$ $= 0 + (V - 0)e^{-t/T}$ $= Ve^{-t/T}$	
		$i = I_f + (I_i - I_f)e^{-t/T}$ $= 0 + (-I - 0)e^{-t/T}$ $= -Ie^{-t/T}$	
	$V_i = -V \quad V_f = 0$ $I_i = V/R = I \quad I_f = 0$ $T = L/R$	$v = V_f + (V_i - V_f)e^{-t/T}$ $= 0 + (-V - 0)e^{-t/T}$ $= -Ve^{-t/T}$	
		$i = I_f + (I_i - I_f)e^{-t/T}$ $= 0 + (I - 0)e^{-t/T}$ $= Ie^{-t/T}$	

Example 9.3

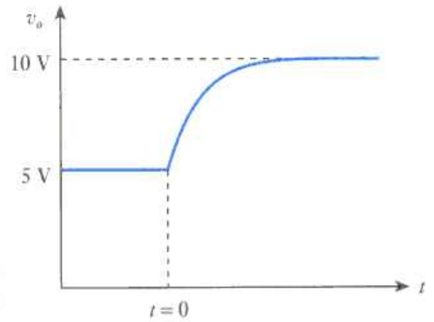
The input voltage to the following *CR* network undergoes a step change from 5 V to 10 V at time $t = 0$. Derive an expression for the resulting output voltage.



In this example the initial value is 5 V and the final value is 10 V. The time constant of the circuit is equal to $CR = 10 \times 10^3 \times 20 \times 10^{-6} = 0.2$ s.

Therefore, from Equation 9.13, for $t \geq 0$

$$v = V_f + (V_i - V_f)e^{-t/\mathrm{T}}$$
$$= 10 + (5 - 10)e^{-t/0.2}$$
$$= 10 - 5e^{-t/0.2} \text{ V}$$



9.4.2 The nature of exponential curves

We have seen that the transients associated with first-order systems contain terms of the form $A(1 - e^{-t/\mathrm{T}})$ or $Ae^{-t/\mathrm{T}}$. The first of these represents a **saturating exponential** waveform and the second a **decaying exponential** waveform. The characteristics of these expressions are shown in Figure 9.5.

In general, one does not need to produce exact plots of such waveforms, but it is useful to know some of their basic properties. Perhaps the most important properties of exponential curves of this form are:

1. The initial slope of the curve crosses the final value of the waveform at a time $t = \mathrm{T}$.
2. At a time $t = \mathrm{T}$, the waveform has achieved approximately 63 per cent of its total transition.
3. The transition is 99 per cent complete after a period of time equal to $5\mathrm{T}$.

9.4.3 Response of first-order systems to pulse and square waveforms

Having looked at the transient response of first-order systems, we are now in a position to consider their response to pulse and square waveforms. Such signals can be viewed as combinations of positive-going and negative-going transitions and can therefore be treated in the same way as the transients

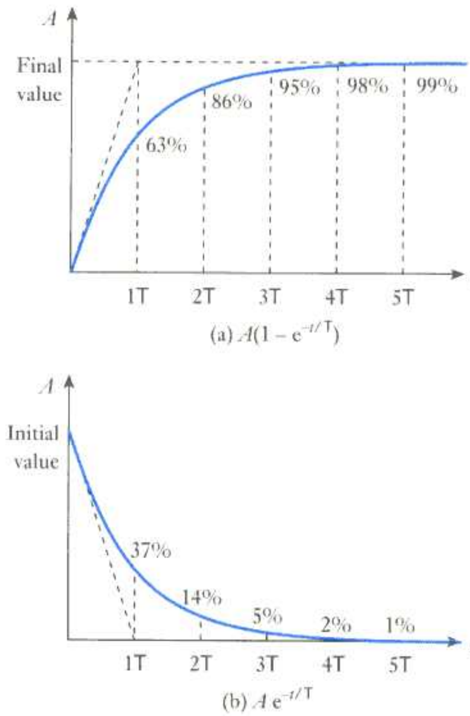


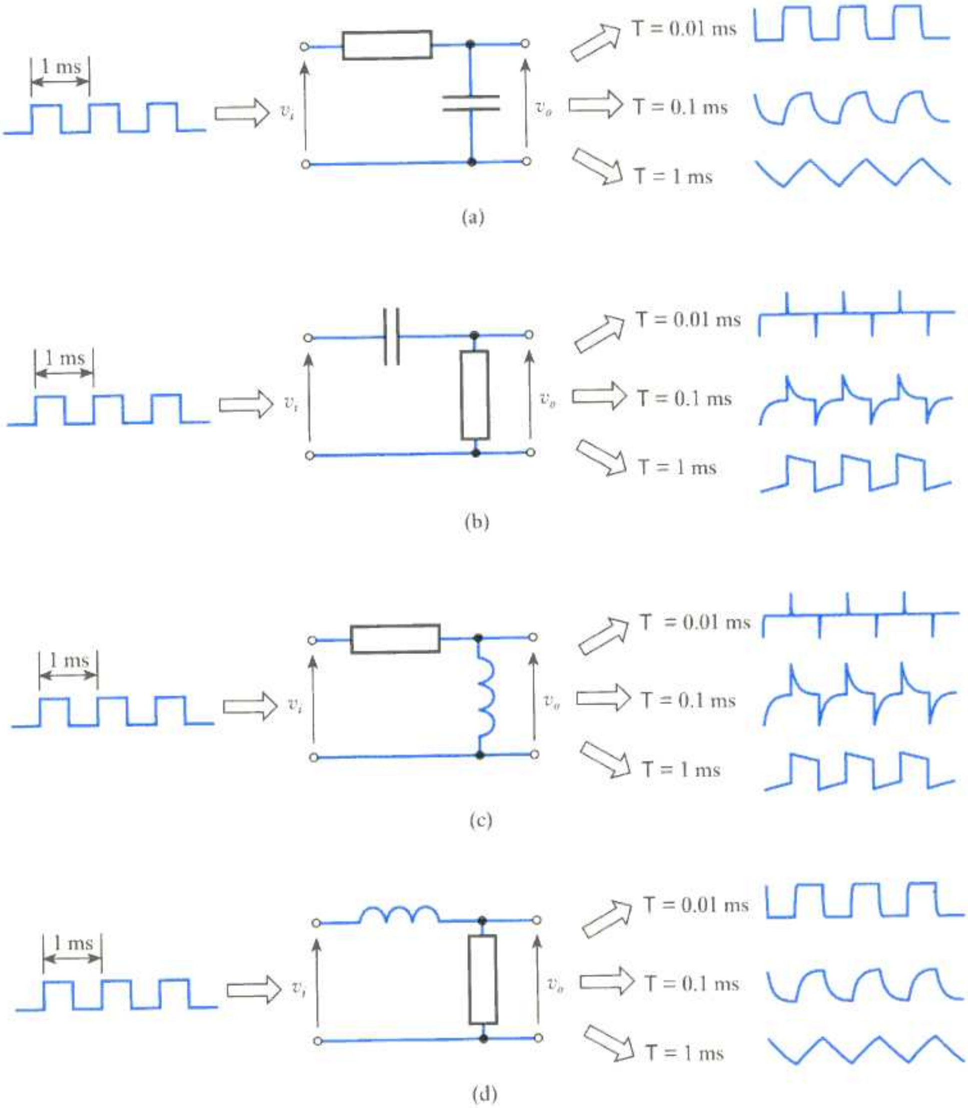
Figure 9.5 Exponential waveforms.

discussed above. This is illustrated in Figure 9.6, which shows how a square waveform of fixed frequency is affected by *RC* and *RL* networks with different time constants.

Figure 9.6(a) shows the action of an *RC* network. We looked at the transient response of such an arrangement in Sections 9.2 and 9.3 and at typical waveforms in Figures 9.1 and 9.3. We noted that the response is exponential, with a rate of change that is determined by the time constant of the circuit. Figure 9.6(a) shows the effect of passing a square wave with a frequency of 1 kHz through *RC* networks with time constants of 0.01 ms, 0.1 ms and 1 ms. The first of these passes the signal with little distortion, since the wavelength of the signal is relatively long compared with the time constant of the circuit. As the time constant is increased to 0.1 ms and then to 1 ms, the distortion becomes more apparent as the network responds more slowly. When the time constant of the *RC* network is large compared with the period of the input waveform, the operation of the circuit resembles that of an **integrator**, and the output represents the integral of the input signal.

Transposing the positions of the resistor and the capacitor in the circuit of Figure 9.6(a) produces the arrangement shown in Figure 9.6(b). The output voltage is now the voltage across the resistor and is therefore proportional to the *current* in the circuit (and hence to the current in the capacitor). We would therefore expect the transients to be similar in shape to the current waveforms shown in Figures 9.1 and 9.3. The steady-state value of the output is zero in this circuit and, when a signal of 1 kHz is passed though a network with a time constant of 0.01 ms, the signal is reduced to a series of spikes. The circuit responds rapidly to the transient change in the input, and the output then decays quickly to its steady-state output value of zero. Here the time constant of the *RL* network is small compared with the period of the input waveform, and the operation of the circuit resembles that of a **differentiator**. As the time constant is increased, the output decays more slowly and the output signal is closer to the input.

Figure 9.6 Response of first-order systems to a square wave.



Figures 9.6(c) and 9.6(d) show first-order *RL* networks and again illustrate the effects of the time constant on the characteristics of the circuits. The pair of circuits produces similar signals to the *RC* circuits (when the configurations are reversed), and again one circuit approximates to an integrator while the other approximates to a differentiator.

File 9A

Computer simulation exercise 9.1

Simulate the circuit of Figure 9.6(a) choosing appropriate component values to produce a time constant of 0.01 ms. Use a digital clock generator to produce a square-wave input signal to this circuit, setting the frequency of the clock to 1 kHz. Observe the output of this circuit and compare this with that predicted in Figure 9.6. Change one of the component values to alter the time constant to 0.1 ms, and then to 1 ms, observing the output in each case. Hence confirm the form of the waveforms shown in the figure. Experiment with both longer and shorter time constants and note the effect on the output.

Repeat this exercise for the remaining three circuits of Figure 9.6.

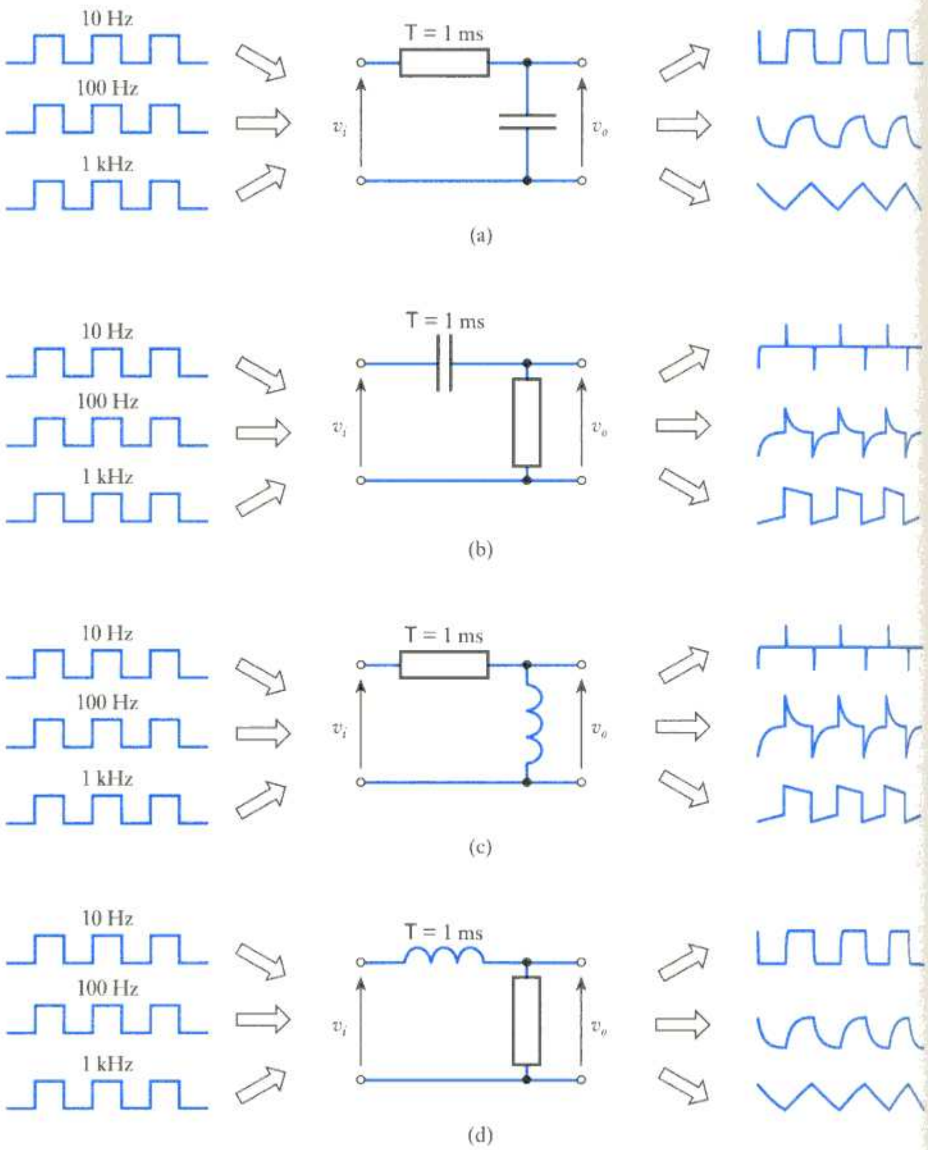


Figure 9.7 Response of first-order systems to square waves of different frequencies.

The shapes of the waveforms in Figure 9.6 are determined by the *relative* values of the time constant of the network and the period of the input waveform. Another way of visualising this relationship is to look at the effect of passing signals of different frequencies through the same network. This is shown in Figure 9.7. Note that the horizontal (time) axis is different in the various waveform plots.

The *RC* network of Figure 9.7(a) is a low-pass filter and therefore low-frequency signals are transmitted with little distortion. However, as the frequency increases the circuit has insufficient time to respond to changes in the input and becomes distorted. At high frequencies, the output resembles the **integral** of the input.

The *RC* network of Figure 9.7(b) is a high-pass filter and therefore high-frequency signals are transmitted with little distortion. At low frequencies, the circuit has plenty of time to respond to changes in the input signal and the output resembles that of a differentiator. As the frequency of the input increases, the network has progressively less time to respond and the output becomes more like the input waveform.

The *RL* network of Figure 9.7(c) represents a high-pass filter and therefore has similar characteristics to those of the *RC* network of Figure 9.7(b). Similarly, the circuit of Figure 9.7(d) is a low-pass filter and behaves in a similar manner to the circuit of Figure 9.7(a).

File 9B

Computer simulation exercise 9.2

Simulate the circuit of Figure 9.7(a) choosing appropriate component values to produce a time constant of 1 ms. Use a digital clock generator to produce a square-wave input signal to this circuit, setting the frequency of the clock to 10 Hz. Observe the output of this circuit and compare this with that predicted in Figure 9.7. Change the frequency of the clock generator to 100 Hz and then to 1 kHz, observing the output in each case. Hence confirm the form of the waveforms shown in the figure. Experiment with both higher and lower frequencies and note the effect on the output.

Repeat this exercise for the remaining three circuits of Figure 9.7.

9.5 Second-order systems

Circuits that contain both capacitance and inductance are normally described by **second-order differential equations** (which may also describe some other circuit configurations). Arrangements described by these equations are termed **second-order systems**. Consider for example the *RLC* circuit of Figure 9.8. Applying Kirchhoff's voltage law to this circuit gives

$$L\frac{di}{dt} + Ri + v_C = V$$

Since i is equal to the current in the capacitor, this is equal to $C\,dv_C/dt$. Differentiating this with respect to t gives $di/dt = C\,d^2v_C/dt^2$, and therefore

$$LC\frac{d^2v_C}{dt^2} + RC\frac{dv_C}{dt} + v_C = V$$

which is a second-order differential equation with constant coefficients.

When a step input is applied to a second-order system, the form of the resultant transient depends on the relative magnitudes of the coefficients of its differential equation. The general form of the differential equation is

$$\frac{1}{\omega_n^2}\frac{d^2y}{dt^2} + \frac{2\zeta}{\omega_n}\frac{dy}{dt} + y = x$$

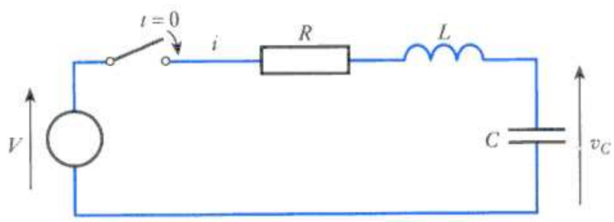


Figure 9.8 A series *RLC* arrangement.

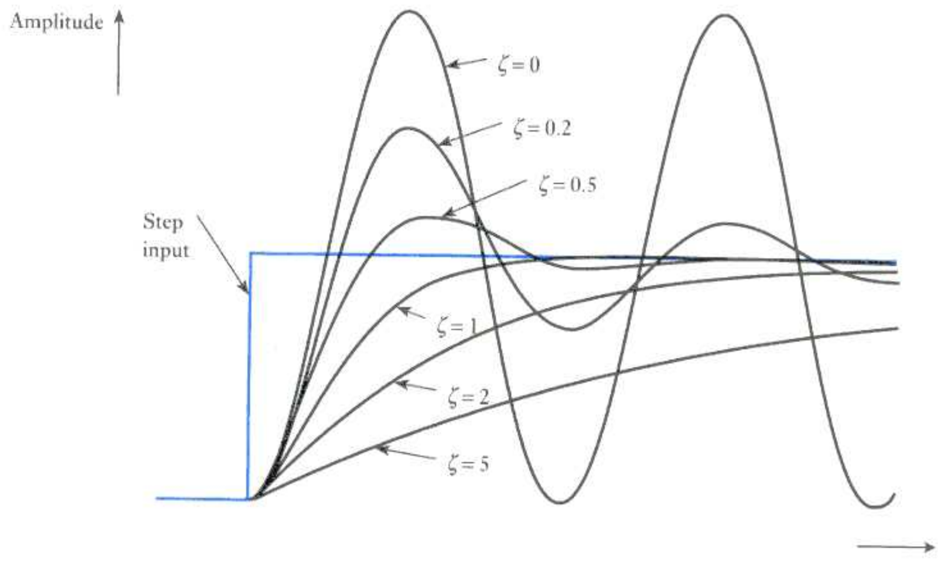


Figure 9.9 Response of second-order systems.

where ω_n is the **undamped natural frequency** in rad/s and ζ (Greek letter *zeta*) is the **damping factor**.

The characteristics of second-order systems with different values of ζ are illustrated in Figure 9.9. This shows the response of such systems to a step change at the input. Small values of the damping factor ζ cause the system to respond more rapidly, but values less than unity cause the system to **overshoot** and oscillate about the final value. When $\zeta = 1$, the system is said to be **critically damped**. This is often the ideal situation for a control system, since this condition produces the fastest response in the absence of overshoot. Values of ζ greater than unity cause the system to be **overdamped**, while values less than unity produce an **underdamped** arrangement. As the damping is reduced, the amount of overshoot produced and the **settling time** both increase. When $\zeta = 0$, the system is said to be **undamped**. This produces a continuous oscillation of the output with a natural frequency of ω_n and a peak height equal to that of the input step.

File 9C

Computer simulation exercise 9.3

Simulate the circuit of Figure 9.8, replacing the voltage source and the switch by a digital clock generator. Use values of 100 Ω, 10 mH and 100 µF for *R*, *L* and *C*, respectively, and set the frequency of the clock generator to 2.5 Hz. Use transient analysis to look at the output voltage over a period of 1 s.

Observe the output of the circuit and note the approximate time taken for the output to change. Increase the value of *R* to 200 Ω and note the effect on the output waveform. Progressively increase *R* up to 1 kΩ and observe the effect.

Now look at the effect of progressively reducing *R* below 100 Ω (down to 1 Ω or less). Estimate from your observations the value of *R* that corresponds to the circuit being critically damped.

9.6 Higher-order systems

Higher-order systems, that is those that are described by third-order, fourth-order or higher-order equations, often have a transient response that is similar to that of the second-order systems described in the last section. Because of the complexity of the mathematics of such systems, they will not be discussed further here.

Key points

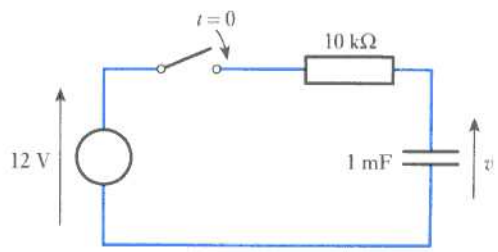
- The reaction of a circuit to instantaneous changes at its input is termed its transient response.
- The charging or discharging of a capacitor, and the energising or de-energising of an inductor, are each associated with exponential voltage and current waveforms.
- Circuits that contain resistance, and either capacitance *or* inductance, may be described by first-order differential equations and are therefore called first-order systems.
- The increasing or decreasing exponential waveforms associated with first-order systems can be found using the initial and final value formulae.
- The transient response of first-order systems can be used to determine their response to both pulse and square waveforms.
- At high frequencies, low-pass networks approximate to integrators.
- At low frequencies, high-pass networks approximate to differentiators.
- Circuits that contain both capacitance and inductance are normally described by second-order differential equations and are termed second-order systems.
- Such systems are characterised by their undamped natural frequency ω_n and their damping factor ζ. The latter determines how rapidly a system responds, while the former dictates the frequency of undamped oscillation.

Exercises

9.1 Explain the meanings of the terms 'steady-state response' and 'transient response'.

9.2 When a voltage is suddenly applied across a series combination of a resistor and an uncharged capacitor, what is the initial current in the circuit? What is the final, or steady-state, current in the circuit?

9.3 The switch in the following circuit is closed at $t = 0$. Derive an expression for the current in the circuit after this time and hence calculate the current in the circuit at $t = 4$ s.



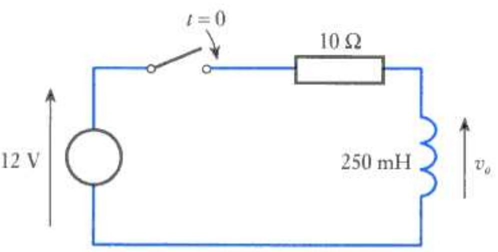
9.4 Simulate the arrangement of Exercise 9.3 and use transient analysis to investigate the current in the circuit. Use a switch element that *closes* at $t = 0$ to

Exercises continued

start the charging process, and use a second switch that *opens* at $t = 0$ to ensure that the capacitor is initially discharged (this second switch should be connected directly across the capacitor). Use your simulation to verify your answer to Exercise 9.3.

9.5 When a voltage is suddenly applied across a series combination of a resistor and an inductor, what is the initial current in the circuit? What is the final, or steady-state, current in the circuit?

9.6 The switch in the following circuit is closed at $t = 0$. Deduce an expression for the output voltage of the circuit and hence calculate the time at which the output voltage will be equal to 8 V.



9.7 Simulate the arrangement of Exercise 9.6 and use transient analysis to investigate the output voltage of the circuit. Use a switch element that closes at $t = 0$ to start the energising process, and use your simulation to verify your answer to Exercise 9.6.

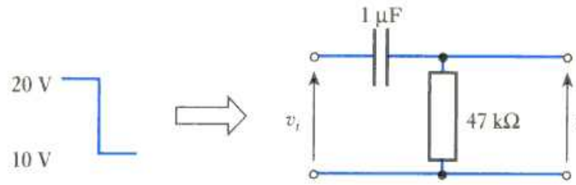
9.8 A capacitor of 25 μF is initially charged to a voltage of 50 V. At time $t = 0$, a resistance of 1 kΩ is connected directly across its terminals. Derive an expression for the voltage across the capacitor as it is discharged and hence determine the time taken for its voltage to drop to 10 V.

9.9 An inductor of 25 mH is passing a current of 1 A. At $t = 0$, the circuit supplying the current is instantly replaced by a resistor of 100 Ω connected directly across the inductor. Derive an expression for the current in the inductor as a function of time and hence determine the time taken for the current to drop to 100 mA.

9.10 What is meant by a 'first-order system', and what kind of circuits fall within this category?

9.11 Explain how the equation for an increasing or decreasing exponential waveform may be found using the initial and final values of the waveform.

9.12 The input voltage to the following *CR* network undergoes a step change from 20 V to 10 V at time $t = 0$. Derive an expression for the resulting output voltage.



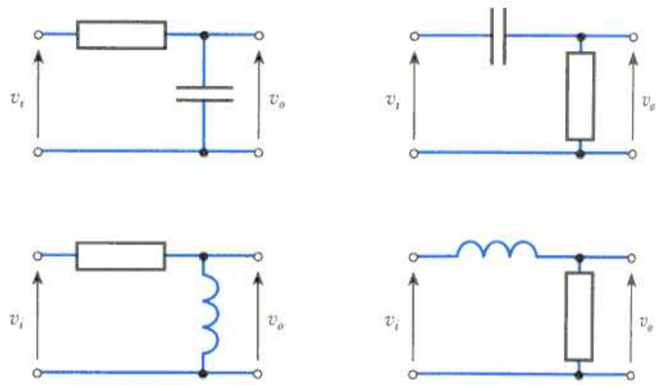
9.13 Sketch the exponential waveform $v = 5e^{-t/10}$.

9.14 For each of the following circuit arrangements, sketch the form of the output voltage when the period of the square-wave input voltage is:

(a) much greater than the time constant of the circuit;

(b) equal to the time constant of the circuit;

(c) much less than the time constant of the circuit.



9.15 Simulate each of the circuit arrangements of Exercise 9.14, selecting component values to give a time constant of 1 ms in each case. Use a digital clock generator to apply a square-wave input voltage to the circuit and use transient analysis to observe the form of the output for input frequencies of 200 Hz, 1 kHz and 5 kHz. Compare these observations with your results for Exercise 9.14.

9.16 Under what circumstances does the behaviour of a first-order high-pass filter resemble that of a differentiator?

9.17 Under what circumstances does the behaviour of a first-order low-pass filter resemble that of an integrator?

9.18 What is meant by a 'second-order system', and what kind of circuits fall within this category?

9.19 Derive an expression for the *current* in the circuit of Figure 9.8.

9.20 Explain what is meant by the terms 'undamped natural frequency' and 'damping factor' as they apply to second-order systems.

9.21 What is meant by 'critical damping' and what value of the damping factor corresponds to this situation?